Representation of BL-algebras with finite and independent spectrum

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Representation theorems for classes of algebras are one of the most important tools to work within the class. They allow to understand the elements of the class in terms of simpler or better known structures, and also to compare different algebras.

For the case of the variety of BL-algebras, the algebraic counterpart of basic logic ([4]), there are representation theorems for the class of finite algebras [3] and for the class of totally ordered algebras (BL-chains, [1]).

As an attempt to extend the representation for BL-chains to the general case, the notion of poset sum is introduced in [5] (called poset product in [2]). With this construction an embedding theorem can be proved, which states that each BL-algebra can be embedded into the poset product of MV-chains and product chains, two particular cases of BL-chains. So each BL-algebra can be seen as a subalgebra of a poset product.

In this talk a representation theorem for a class of BL-algebras will be presented. These are algebras with finite and independent prime spectrum. Such class properly includes the class of finite algebras. The idea of independent spectrum will be explained during the talk.

The representation relies on the decomposition theorem for BL-chains that states that each nontrivial BL-chain can be uniquely decomposed as an ordinal sum of nontrivial totally ordered Wajsberg hoops. We will show that a BL-algebra $\mathbf{A}$ with finite and independent spectrum P is isomorphic to an algebra of functions $R(\mathbf{A})$. For each $p \in P$ we define a totally ordered Wajsberg hoop $\mathbf{W}_p$. The domain of each function f of $R(\mathbf{A})$ is a subset $P_f \subseteq P$ and $f(p) \in \mathbf{W}_p$.

The proof of the representation takes into account the embedding of each BL-algebra into a poset product.

References

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