

Applications of the semisimple tensor product of MV-algebras

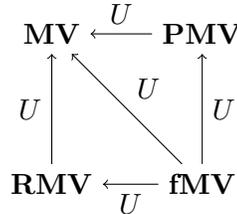
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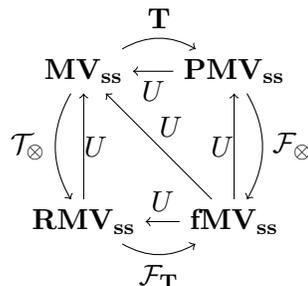
Joint work with S. Lapenta.

Since the real interval $[0, 1]$ is closed to the product operation, a natural problem was to find a complete axiomatization for the variety generated by the standard MV-algebra $([0, 1], \oplus, \neg, 0)$ endowed with the real product. If the product operation is defined as a bilinear function $\cdot : [0, 1] \times [0, 1] \rightarrow [0, 1]$ then the standard model is $([0, 1], \oplus, \cdot, \neg, 0)$; if the product is a scalar multiplication then the standard model is $([0, 1], \oplus, \{r \mid r \in [0, 1]\}, \neg, 0)$, where the function $x \mapsto rx$ is a linear for any $r \in [0, 1]$. The approach based on the internal binary product led to the notion of *PMV-algebra* [1, 6, 7], while the approach based on the scalar multiplication led to the notion of *Riesz MV-algebra* [2]. In [4] we defined the *fMV-algebras* adding both an internal product and a scalar multiplication; in this case, our standard model is $([0, 1], \oplus, \cdot, \{r \mid r \in [0, 1]\}, \neg, 0)$. The varieties of MV-algebras and Riesz MV-algebras are generated by their corresponding standard models; this is not the case for PMV-algebras and fMV-algebras: in these classes the standard models generate proper sub-varieties.

There are obvious forgetful functors between the above mentioned classes of structures:



Our goal is to define the left adjoint functors. We do this in [3] for the corresponding subclasses of semisimple structures using the semisimple MV-algebraic tensor product defined in [8] and its scalar extension property [5]. Note that all algebras with product are considered unital. Of particular importance is the *semisimple tensor PMV-algebra of an MV-algebra*, whose construction is inspired by the well-known definition of the tensor algebra. The main result asserts that the following diagram is commutative:



As consequence we prove the amalgamation property for semisimple PMV-algebras, semisimple Riesz MV-algebras and semisimple fMV-algebras.

References

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