Applications of the semisimple tensor product of MV-algebras

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Since the real interval $[0, 1]$ is closed to the product operation, a natural problem was to find a complete axiomatization for the variety generated by the standard MV-algebra $([0, 1], \oplus, \neg, 0)$ endowed with the real product. If the product operation is defined as a bilinear function $\cdot : [0, 1] \times [0, 1] \to [0, 1]$ then the standard model is $([0, 1], \oplus, \cdot, \neg, 0)$; if the product is a scalar multiplication then the standard model is $([0, 1], \oplus, \{r | r \in [0, 1]\}, \neg, 0)$, where the function $x \mapsto rx$ is a linear for any $r \in [0, 1]$. The approach based on the internal binary product led to the notion of PMV-algebra [1, 6, 7], while the approach based on the scalar multiplication led to the notion of Riesz MV-algebra [2]. In [4] we defined the $fMV$-algebras adding both an internal product and a scalar multiplication; in this case, our standard model is $([0, 1], \oplus, \cdot, \{r | r \in [0, 1]\}, \neg, 0)$. The varieties of MV-algebras and Riesz MV-algebras are generated by their corresponding standard models; this is not the case for PMV-algebras and $fMV$-algebras: in these classes the standard models generate proper sub-varieties.

There are obvious forgetful functors between the above mentioned classes of structures:

$$
\begin{array}{ccc}
\text{MV} & \xrightarrow{U} & \text{PMV} \\
\text{RMV} & \xrightarrow{U} & \text{fMV} \\
\end{array}
$$

Our goal is to define the left adjoint functors. We do this in [3] for the corresponding subclasses of semisimple structures using the semisimple MV-algebraic tensor product defined in [8] and its scalar extension property [5]. Note that all algebras with product are considered unital. Of particular importance is the semisimple tensor $PMV$-algebra of an MV-algebra, whose construction is inspired by the well-known definition of the tensor algebra. The main result asserts that the following diagram is commutative:

$$
\begin{array}{ccc}
\text{MV}_{ss} & \xrightarrow{T} & \text{PMV}_{ss} \\
\text{RMV}_{ss} & \xrightarrow{T} & \text{fMV}_{ss} \\
\end{array}
$$

As consequence we prove the amalgamation property for semisimple PMV-algebras, semisimple Riesz MV-algebras and semisimple $fMV$-algebras.
References


