

Variants of Ulam game and game semantics for many-valued logics

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As shown by Mundici [Mu1], the Rényi-Ulam game (that is, the Ulam game with a maximum number of lies) is a very interesting game, because it has applications to the treatment of uncertainty and at the same time it constitutes a sound and complete game semantics for Łukasiewicz logic. The idea is that in the Ulam game with $\leq e$ lies any sequence σ of questions-answers is coded by a truth function f_σ from the search space S into $\left\{0, \frac{1}{e+1}, \frac{2}{e+1}, \dots, \frac{e}{e+1}, 1\right\}$, and the juxtaposition of two sequences σ and τ is coded by the Łukasiewicz conjunction of f_σ and f_τ . Moreover it is possible to represent the other connectives of Łukasiewicz logic in terms of operations on truth functions.

In [CMBL] the authors investigate a multichannel variant of the Ulam game, and prove that this variant constitutes a complete game semantics for Hájek's logic BL. This variant admits the Rényi Ulam game as a special case. However, although both Gödel Logic and Product Logic extend BL, the game proposed by Cicalese and Mundici does not provide a game semantics for these logics. The goal of this talk is to present a complete game semantics for BL which includes a complete game semantics for Łukasiewicz, Gödel and Product Logics as special cases.

We start from the case of Gödel Logic. In this case lies are not admitted, but the answers can use different channels, and each channel has a fixed probability to work correctly. Moreover if a channel C works correctly, then all the answers using channel C will be correct, otherwise the answers may contain arbitrarily many lies.

In the case of Product Logic, we combine a probabilistic variant introduced by Pelc [P], in which Responder may lie with a fixed probability, and a variant by Cicalese and Mundici [CM], in which any answer YES must be correct, while an answer NO might be incorrect.

Finally, we consider a variant of the game semantics proposed in [CMBL], which not only constitutes a complete game semantics for BL, but avoids the requirement on increasingly noisy channels and admits semantics for Gödel and Product Logics as special cases.

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