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Exchange Market Pressure:
Some Caveats
in Empirical Applications

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EXCHANGE MARKET PRESSURE: SOME CAVEATS IN EMPIRICAL APPLICATIONS¹

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The Exchange Market Pressure (EMP) Index, developed by Eichengreen et al. [1994], is widely used to study currency crises as a tool to signal whether pressures on a currency are softened or warded off through monetary authorities' interventions or whether a currency crisis has originated. In this paper we show how the index is sensitive to some assumptions behind the aggregation of the information available (exchange rates, interest rates and reserves), especially when emerging countries are involved. Specifically, we address the way exchange rate variations are computed and the impact of different definitions of the reserves, and we question the constancy of the weights adopted. These issues compound with the choice of a fixed threshold when crisis episodes are identified through EMP. As a result, the dichotomous crisis variable thus derived when adopted as a dependent variable may lead to varied results in subsequent econometric analysis.

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1. Introduction

Currency crises in emerging and developing economies have been extensively analyzed in the literature with a variety of analytical tools proposed to identify crisis episodes. One of the indexes that are most widely adopted to signal the break up of a crisis is the Exchange Market Pressure, EMP hereafter, introduced in a seminal paper by Girton and Roper [1977] to investigate independence of and interventionist stance by a Central Bank as a simple average of exchange rate changes and a foreign reserve depletion indicator. An extension was suggested by Eichengreen, Rose and Wyplosz [1994 – henceforth ERW], adding the interest rate spread to the index, to describe possible monetary policy responses to a disequilibrium in the foreign exchange market. Their index is a weighted average to take into consideration the different variability in the three variables.

To the best of our knowledge, ERW were the first to employ the EMP as a basis for the analysis of currency crises: in their application on exchange market behaviour for developed countries, when the EMP passes over a threshold, excess pressure is flagged and a binary variable takes on a value of one.

The EMP index is meant to capture depreciations but also the type of pressure on a currency (as would happen in the presence of depreciation expectations) which is softened or diverted through monetary authority interventions, and does not necessarily show up in the observed behaviour of nominal exchange rate dynamics (i.e. 'Peso Problem' type, Evans [1996]) . In this view, crisis episodes occur even if speculative attacks are not successful.

Other papers have used the index (e.g. Eichengreen et al. [1996], Tudela [2004]), at times defined differently and with a different threshold to define a crisis. Some (e.g Sachs et al. [1996]) limit themselves to a two-component version, which excludes interest rates. Kaminsky et al [1998] and Kaminsky and Reinhart [1999] do the same but on the ground of data limitations, while Tanner [2001] has more theoretical objections to the insertion of what she sees as a response variable rather than an indicator.

This paper focuses on the methodological issues related to the use of the EMP index, as this has found a wide adoption in the literature on currency crises [Eichengreen et al 1994 and 1996; Kaminsky and Reinhart 1998; Sachs et al. 1996; Tanner 2001; Tudela 2004]. Without questioning the general theoretical framework within which the index is derived, we point out how seemingly harmless - and often hidden - choices required in the aggregation of the components of the index, may affect the results of subsequent empirical analysis. EMP may thus suffer from some weaknesses that cast doubts on its reliability as a basis for econometric analysis, especially when emerging countries are involved.

Several issues related to the adoption of the EMP index will be described in this paper. In the second section we discuss the way in which the EMP is built, pointing out the statistical issues that emerge with the use of an index based on multi-dimensional information. In the third section, we highlight the *ad hoc* assumptions introduced to build on the EMP a binary crisis variable and the limits of a parametric definition of crisis². In a preliminary attempt to show how methodological choices do matter and are able to affect the econometric analysis on currency crises determinants, we employ data from a sample of 26 countries to show how the EMP index and the subsequent crisis indicator vary with different choices available.³ It is important to consider emerging countries alike, since the sensitivity of the EMP index proves to be directly related to the specific economic characteristics of a country. Finally, it is worth noting that when developing countries are involved, the choice of a suitable benchmark is not neutral. Pontines and Siregar [2004a] have already showed that if Japan replaces the US as the reference country, the EMP for the Thai baht changes significantly, along with its capacity to signal the actual crisis episodes that hit the East Asian country in the late 90s. A brief discussion on this issue and a generalization on how the EMP is affected changing the reference country will be argued in the fourth section.

² We will not discuss the recent papers by Pontines and Siregar [2004b]. They avoid the problem of a parametric definition of the index using the Extreme Value Theory.

³ See the Appendix A1 for a description of the dataset. The sample is made up of ten European countries (Denmark, Finland, France, Germany, Italy, the Netherlands, Spain, Sweden, Turkey and the United Kingdom), seven Latin American (Argentina, Chile, Colombia, Mexico, Peru, Uruguay and Venezuela), seven Asian (Indonesia, Japan, Malaysia, Philippines, Singapore, South Korea and Thailand), the United States of America and Canada.

2. The Exchange Market Pressure index

The EMP proposed by ERW is defined as:

$$EMP_t = \alpha \Delta_{\%} e_t + \beta \Delta(i_t - i_t^f) + \gamma \Delta(r_t^f - r_t) \quad [1]$$

that is a weighted average (with positive weights α , β and γ) of three variables: $\Delta_{\%} e_t$ is the percentage change in the nominal exchange rate against a reference currency, $\Delta(i_t - i_t^f)$ represents the variation in the spread between the domestic interest rate and the foreign interest rate and finally $\Delta(r_t^f - r_t)$ is the change in the spread between foreign reserves (relative to monetary base) abroad and at home. The EMP index can take values on the real line, with high positive values associated to a pressure on the domestic currency, as a combination of a nominal depreciation, a widening of the interest rate spread or a loss of foreign reserves.

To avoid an overlapping of the issues that arise from the construction of the EMP itself with those that emerge from the use of the EMP as crises indicator we will focus on the former in this section and devote the next section to the latter. Particularly, we will first discuss how three different issues influence the index: the choice of the weights; the use of logarithmic difference to approximate the percentage change of the exchange rate; the inclusion of gross or net foreign reserves. The EMP turns out to be notably sensitive to seemingly minor choices, and this poses the problem of its *fine tuning*, as it is not always possible to rank possible alternatives, and different choices can be defended on different empirical or theoretical grounds.⁴

2.1 Weighting

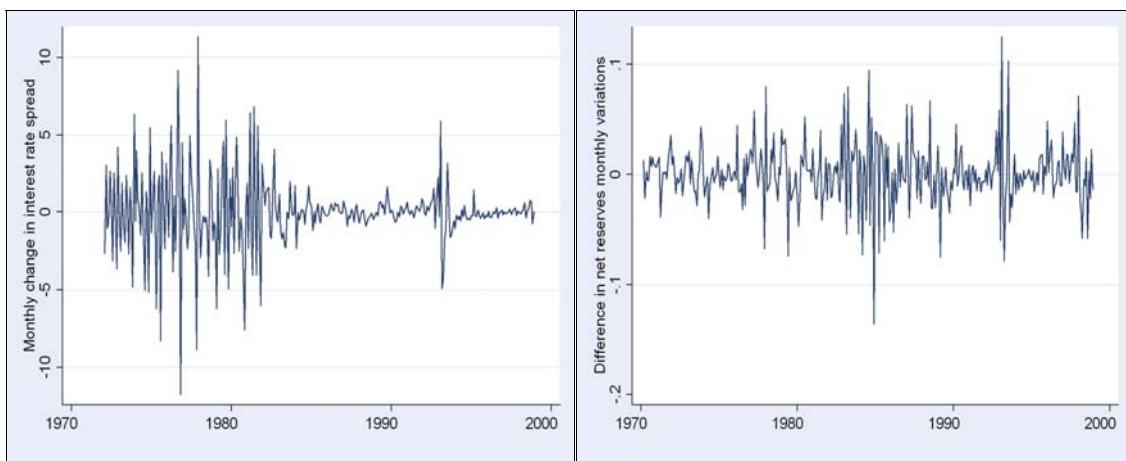
Since the three variables underlying the EMP are usually characterized by different volatilities, their aggregation has to be conducted in such a way that prevents the most volatile component from

⁴ In the next subsections, we will identify these choices and provide illustrative examples of their bearings on the index, even though a fuller understanding of their empirical relevance can be more easily described once the index is used to build a binary indicator of crisis periods, and thus we let this to a later section.

dominating the whole index. Excluding Argentina, the standard deviation of exchange rate variations and of the interest rate differential has a sample average of approximately 1.5 and 900 times that of the foreign reserves movements. ERW suggest standardizing the three components, i.e. replacing each of the weights that appear in [1] with the reciprocal of the country-specific standard deviation of the relative series, in order to equalize unconditional volatilities.

However, financial time series are characterized by volatility clustering (Engle, 2005) and constant weights do not allow to adequately smooth volatility when this is time varying or undergoes structural breaks.⁵ Figure [1] reports the monthly change in the interest rate and foreign reserves over monetary base spread between Denmark and the US in the period 1970-2002. In the early 80s, a structural shift in the volatility of the interest rate spread is observed, and another high volatility cluster is recorded in the mid 90s. The standardization of interest rate spread through its sample variance implies that the evolution of the series does not convey much information to the overall index during periods of low volatility.

Figure 1. Monthly variations in the interest rate spread (left) and in foreign reserves over the monetary base (right), Denmark, 1970-2000.



⁵ Moreover, the reliance on sample variances implies that the weight sets are sensitive to the presence of outliers in the series. Argentina has been excluded from the computation of sample averages of the standard deviation as its domestic interest rate in 1990 skyrocketed to 4,670,000 percent in February and further increased to 6,970,000 percent in March, in a desperate effort to stop the free fall of the peso. These two spikes increase the standard deviation of the interest rate differentials, reducing its weight virtually to zero. Thus, the EMP for Argentina reduces to a two-component version, with movements in the interest rate failing to convey any information but in February and March 1990.

The volatility clustering of the different series could be consistent with changes over time of the preferred instruments that are chosen by the monetary authorities in order to face pressure on the domestic currency. The figure on the right reveals how the reduction in the volatility of interest rate spread for Denmark was followed by an increase in the volatility of the ratio of international reserves over the monetary base. Possible attempts to overcome the tight limits of time-invariant standardization to rely either on moving standard deviations or to compute sample variances over sub-intervals of the time frame of the analysis, i.e. five or ten years standard deviations.

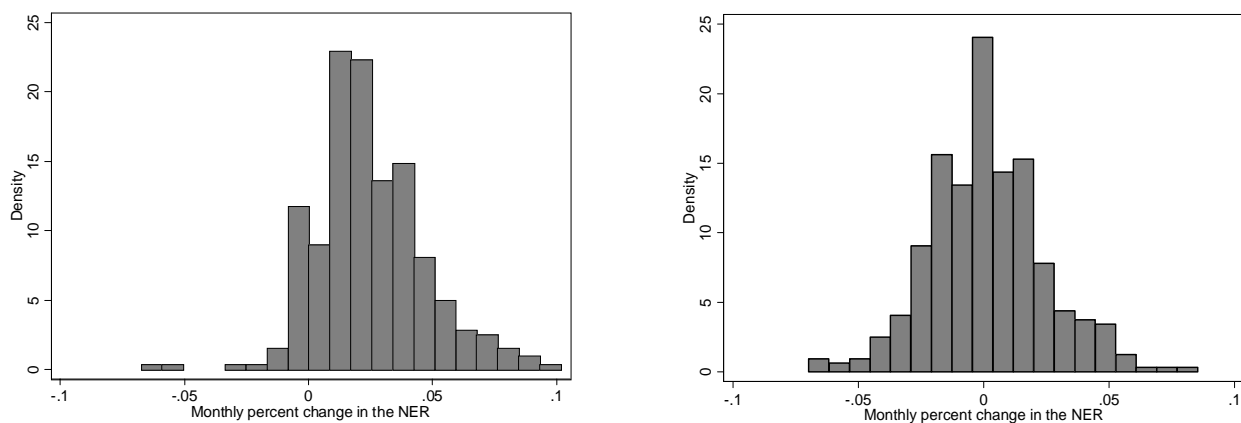
2.2 Exchange rate variations

The first component of the EMP that appears in [1] is given by the rate of change of the nominal exchange rate of the domestic currency against an anchor currency over the period of reference. With a few exceptions, the method used to compute the rate of variation is left unspecified.⁶ The logarithmic difference provides a good approximation of the rate of change around zero, and hence its use is rather immaterial as far as this choice refers to developed countries. But the structural differences that characterize the distribution of the monthly rate of variations of the nominal value of developed and developing countries' currencies imply that the method of computation does matter when developing countries are involved. An enduring tendency towards depreciation, coupled with periods of sudden and sharp loss of value, is mostly found when looking at the evolution of the nominal exchange rate between a developing country's and a developed country's currency in the long run. This trend may be due to several underlying factors, most notably the existence of a positive inflation differential with the reference country. On the other hand, the nominal exchange rate between a pair of developed countries tends often to be fairly stable if a sufficiently long time span is considered, and monthly variations tend to be smaller in absolute value.

⁶ Eichengreen et al. [1996] employ logarithmic differences, while Pontines and Siregar [2004] use the exact formula..

Figure [2] presents the distribution of the monthly rate of variation of nominal exchange rate with the US dollar for the Uruguayan peso and the British pound, to provide a telling description of the structural differences that we have just mentioned. The distribution of the monthly rate of change of the exchange rate between the pound and the US dollar has a mean of 0.13 percent, and it is fairly symmetric around zero. On the other hand, the distribution of the rate of change of the Uruguayan peso against the US dollar has a mean equal to 5.28 percent; it is asymmetric, with 94.92 percent of the observations signalling depreciation, and with a thicker upper tail. For the pound, just one observation lies above ten percent (September 1992), i.e. outside the range that is shown in the plot, while for the peso we find ten observations above that threshold.

Figure [2] Distribution of monthly rate of variation of nominal exchange rate with the US dollar for the Uruguayan peso (left) and the British pound (right), 1970-2002



The relevance of the method used to compute the variation of the exchange rate is twofold: first, logs provide a poor approximation of the rate of change when this is large; second, and most notably, the logarithmic approximation determines a significantly lower *variance* of the sample distribution of the rate of variation of the nominal exchange rate for most developing countries. *A priori*, it is not possible to predict whether the use of logarithms will generate a distribution with a higher or smaller variance, as the logarithmic function is a contraction mapping on the domain $(1, +\infty)$ and an expansion mapping in the domain $(0, 1)$.⁷ But, if the sample values of the rate of change lie disproportionately above or below zero, the use of the logarithmic approximation influences the

⁷ Appendix A2 formally demonstrates that the transformation of a series through a contraction mapping reduces its variance.

standard deviation of the transformed values in a predictable way.⁸ For the Uruguayan peso, the standard deviation of its monthly rate of change is 0.47 when the rate is computed with the exact formula and 0.17 when logs are applied. For the British pound we need a four-digit precision to appreciate a difference between the two methods, as the standard deviation decreases from 0.0242 to 0.0241.

Table [1] shows how the method of computation of the exchange rate variations affects the corresponding standard deviation for the sample countries. The difference is negligible for developed countries, while it shows up as systematic for emerging countries. The most notable differences are reported for Latin American countries. Many of these countries experienced hyperinflation in the '80s and in the '90s, and this caused sharp and prolonged devaluations of their currencies. For Chile, Peru and Uruguay, the use of logarithms reduces the standard deviation of the exchange rate variable to less than *half* of the value that is found when rates of variation are computed with the exact formula.

Table [1] Effect of the logarithmic transformation on the standard deviation of the nominal exchange rate variations

	Standard deviation s_e of the monthly rate of change of the exchange rate		
	<i>ratio (percent)</i>	<i>ratio (percent)</i>	
Developed Countries	99.583	Emerging Countries	
		71.564	
Denmark	100.00	Argentina	60.60
Finland	99.16	Chile	44.81
France	99.23	Colombia	99.34
Germany	100.37	Mexico	85.74
Italy	99.19	Peru	37.34
Netherlands	100.00	Uruguay	36.59
Spain	98.80	Venezuela	69.99
Sweden	98.35	Turkey	81.94
United Kingdom	99.59	Indonesia	82.96
Canada	100.00	Korea	89.37
Japan	100.72	Malaysia	98.52
		Philippines	88.69
		Singapore	100.00
		Thailand	96.51

⁸ Denoting $x_t = \frac{e_t - e_{t-1}}{e_{t-1}}$ and $x_t' = \ln\left(\frac{e_t}{e_{t-1}}\right)$, we have that $x_t' = \ln(x_t + 1)$.

2.3 International reserves

The inclusion of international reserves in the EMP is intended to capture those speculative pressures that lead the central bank to intervene on the foreign exchange spot market in defence of its currency. However, just a part of central bank's operations translate into a variation in the level of international reserves. A central bank can attempt to defend its currency by drawing on stand-by credits or through off-balance-sheet transactions, as forward market interventions, thus not committing any of its foreign assets on the spot market. Thailand's gross international reserves had remained fairly stable in the first six months of 1997, but a severe currency crisis broke out on the 2nd of July, with a large devaluation of the *baht*, when it became apparent that all of its foreign assets had already been committed on the forward market. This episode shows clearly the limitations of inferring the magnitude of monetary authorities' interventions on foreign exchange markets from variations in the level of reserves, but this remains the only feasible option as central banks do not give notice of their off-balance-sheet operations.

Data availability is a strong constraint, at times, in what concerns a central bank's foreign assets since they may be reported either net or gross of international liabilities. Choosing one or the other, or even mixing the two measures in a cross country context is by no means neutral as differences can be rather substantial and the movements of gross and net reserves sometimes can convey conflicting information, thus leading to different perceptions of the pressure a currency is subject to. The inclusion of gross reserves may be preferable if one is interested in liquidity crises, while net reserves may be better suited for studies on the determinants of solvency crises.

If the choice between gross and net reserves is not guided by a clear cut definition of financial crisis, this should be influenced by the need to treat in a consistent way the countries that are included in the sample. Countries may differ with respect to the frequency with which they activate credit lines in foreign currencies, and the same country can change over time its reliance on off-balance-sheet operations to face speculative pressure on its currency. When gross reserves are

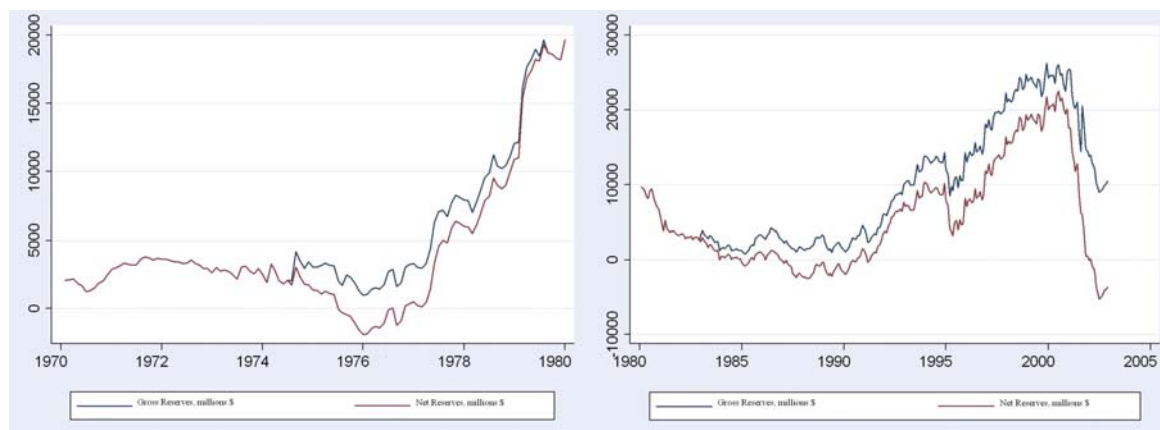
chosen as a component of the EMP index, the index will reveal, *ceteris paribus*, a softer pressure on a currency whose exchange rate is backed by the activation of credit lines in foreign currencies.⁹ If the monetary authorities can draw resources from these credit lines, they can face speculative pressure on their currency without having to deplete their gross foreign assets. In particular, countries can activate a stand-by arrangement or a loan with the International Monetary Fund when they experience Balance of Payments need. The amount of *Fund credit and loans outstanding* is reported for each member country by the IFS, and this series – that is published on a monthly basis by the IMF - can be used to account for a specific form of international liabilities that is extremely significant for developing countries.¹⁰

To provide a concrete example, Figure [3] shows the evolution of international reserves – both gross and net of the liabilities towards the IMF – for Argentina over the period 1993-2002 and for Italy in the 1970s. The figure for Argentina reveals a prolonged period of Fund assistance: it is interesting to focus on the crisis of 2001. While the pattern followed by net reserves signals a much greater incidence of the crisis than what be inferred looking at the evolution of gross reserves. The inclusion of either series in the EMP will thus provide a different picture of the recent Argentinean crisis.

⁹ The choice between net and gross reserves implies a change in the weights, if the weights of the three components of the index are chosen so to equalize their conditional volatility. However, from our sample it does not emerge any systematic difference in the volatility of gross or net reserves, and changes in the weights are rather small.

¹⁰ This series does not include the undrawn balance of a stand-by agreement with the IMF. In principle, this voice could be considered, if this contributed to have a fuller measure of the capacity of a central bank to defend its currency through interventions on the foreign exchange market. But, as Mussa and Savastano [2001] point out, “*more than a third of all Fund arrangements approved between 1973 and 1997 ended with a disbursement of less than an half of the initially agreed support. (...) Mainly these were cases where the program went off track because policies deviated significantly from those agreed with the IMF and subsequent negotiations failed to reach agreement on a modified program.*” This reflects the strong conditionality of Fund credit, which does not allow considering the undrawn balance of a stand-by agreement with the IMF as resources that can be freely used by a central bank to defend its currency.

Figure [3] Foreign Reserves, Gross and Net of liabilities towards the IMF (millions of dollars),
Italy 1970-1980 (left) and Argentina 1980-2002 (right)



When international reserves are considered net of monetary authorities' liabilities towards the IMF, this allows for the inclusion in the same sample of countries that receive significant financial assistance from the IMF before or in the aftermath of a crisis, and countries that do not. The different paths followed by Malaysia and South Korea in 1997, after the crisis had broken out in Thailand, constitute a good example of the need for a consistent treatment of international reserves variations for countries that have to be included in the same sample. As it is well known, Malaysia pursued its own adjustment program, introducing temporary restrictions to capital movements, without drawing resources from the IMF. On the other hand, South Korea received 11,014 millions of US dollars in December 1997, and its debt towards the IMF kept on rising till October 1998, when it reached 18,754 millions, well above 1,600 percent of its Fund quota. The resources South Korea received from the Fund in December 1997 amounted to more than an half of its gross international reserves.

If reserves are considered gross of central bank's liabilities towards the IMF, then there is the risk of underestimating the pressure a currency is subject to, when its reserves are supported by large inflows of Fund credit. Even though this point is clearly relevant for developing countries, it should be noted that if the time span of the analysis covers the 70s, the need to account for liabilities towards the Fund concerns developed countries as well, as Figure [3] signals. In the 70s and early 80s, several European countries received credit from the IMF, often well above their respective

Fund quotas. For example, in January 1976, Italian liabilities towards the IMF were almost *three times* the level of Italian gross foreign reserves, and thus its net reserves were negative.

3. EMP-based definitions of currency crises

A widely adopted approach to the analysis of currency crises entails the division of sample observations among crisis episodes and periods of tranquillity. This requires an identification rule: the crises could be defined either as the transition between two different exchange rate regimes, as the flotation of a pegged exchange rate, or as exceptional movements of the nominal exchange rate [Frankel and Rose, 1996] or of a broader index of excess demand for foreign currency, as the EMP¹¹.

In ERW and Kaminsky and Reinhart [1999] a crisis is signalled when the EMP is above a critical threshold defined as a function of the (country-specific) sample mean and standard deviation of the index. Hence, their index (IC) reveals a currency crisis when the *standardized* EMP index crosses a threshold τ :

$$\begin{cases} IC_t = 1 \text{ if } \frac{EMP_t - \mu_{EMP}}{\sigma_{EMP}} > \tau \\ IC_t = 0 \text{ if } \frac{EMP_t - \mu_{EMP}}{\sigma_{EMP}} \leq \tau \end{cases} \quad [3]$$

The definition of the critical threshold is a discretionary “rule of thumb” for the identification of a currency crisis, and the *crisis set*¹², i.e. the set of observations that are identified as crisis episodes, is clearly sensitive to the choice of the width of the tranquillity band. ERW set τ equal to 1.5, while Kaminsky *et al.* [1998] and Kaminsky and Reinhart [1999] adopt a critical value of 3. These

¹¹ See Bubula and Otker-Robe [2003] for an overview of identification rules proposed in the literature.

¹² Formally, the crisis set IC_i for a single country identified and specification i of EMP index and crisis identification rule is defined as:

$$IC_i = \left\{ t \in T \mid \frac{EMP_t - \mu_{EMP}}{\sigma_{EMP}} > \tau \right\} \text{ where } T \text{ is the horizon of the analysis.}$$

thresholds seem to be inspired by some sort of parametric assumption on the distribution of the standardized EMP. As noted by Pontines and Siregar [2004a, 2004b] this may be a strong limitation of the procedure as non-normality is to be expected. The *crisis set* is thus dependent on the adopted identification rule, and this suggests the opportunity to gather some indications from the previous discussion on the construction of the EMP and undertake some sensitivity analysis on the definition of a crisis.

Adopting the definition of the EMP index and the crisis identification rule from ERW,¹³ we can compare the effects of alternative specifications for the items discussed above (exchange rate computation, gross or net foreign reserves, weighting). A simple index of the distance between outcomes will help us in appraising divergence of the results.

3.1 Sensitivity Analysis for the fine tuning issues

Let us consider the classification of each period in the sample as a crisis or a tranquil period according to the specification adopted, and let us define a divergence index $Div_{i,j}$ between two different specifications i and j :

$$Div_{i,j} = 1 - \frac{\#(IC_i \cap IC_j)}{\#(IC_i \cup IC_j)} \quad [4]$$

that is, 1 minus the ratio of the number of periods classified as a crisis by both specifications to the total number of crises identified by either specification. This measure ranges between 0 (perfect coincidence of the two crisis sets) and 1 (no common identification). Table [2] displays the extent of divergence when a) the rate of exchange rate variation is computed exactly (0.01 average divergence for developed countries versus 0.10 for emerging countries), b) the reserves are taken

¹³ The rate of variation of the exchange rate is computed with the log approximation, international reserves are taken gross of any liabilities, Germany is the reference country for all European countries except the United Kingdom while the US is the reference country for the remaining countries. The weights of the three components are time invariant, the average and standard deviation of the EMP index are computed over the whole sample and finally τ is set equal to 1.5

net of liabilities towards the IMF (0.01 versus 0.16), and c) the three components are standardized through 11-year moving standard deviations (0.24 versus 0.34).

Looking at developed countries, it is apparent that the way exchange rates and reserves are computed is irrelevant: the logarithmic difference provides a close approximation of the actual rate of variation of the exchange rate of their currencies, and the use of either gross or of net reserves in ERW does not significantly affect the identification of a crisis period.¹⁴ This is not true for emerging countries: hyperinflation episodes in Latin America imply that logarithms provide a poor approximation of the sudden and wide depreciations of the domestic currencies they induced. Moreover, the relevance of the choice between either gross or net reserves extends to Asian countries as well, especially to Indonesia, Korea and Thailand, as they draw heavily on IMF credit. As noted above, the largest overall changes come from time-varying weights which affect developed countries as well with highest values for Sweden (0.44), Italy (0.30) and the United Kingdom (0.27).

¹⁴ A minor exception is United Kingdom, that, as noted earlier, in the 1970s had periods of *negative* net reserves.

Table [2] Degree of divergence across specifications by indicator (formula [4])

Country	Exchange Rate (Log vs. Exact Formula)	Reserves (Gross vs. Net)	Weights (Constant vs. Time-varying)
Developed	0.01	0.01	0.24
Denmark	0.00	0.00	0.25
Finland	0.00	0.00	0.14
France	0.00	0.00	0.21
Germany	0.00	0.00	0.25
Italy	0.00	0.00	0.30
Netherlands	0.00	0.00	0.23
Spain	0.06	0.00	0.16
Sweden	0.00	0.00	0.44
United Kingdom	0.03	0.07	0.27
Canada	0.00	0.00	0.14
Japan	0.00	0.00	0.16
Emerging	0.10	0.16	0.34
Argentina	0.20	0.38	0.38
Chile	0.38	0.14	0.59
Colombia	0.07	0.00	0.19
Mexico	0.05	0.28	0.25
Peru	0.33	0.31	0.76
Uruguay	0.31	0.23	0.41
Venezuela	0.00	0.25	0.20
Turkey	0.00	0.06	0.22
Indonesia	0.13	0.13	0.13
Korea	0.06	0.16	0.38
Malaysia	0.00	0.00	0.33
Philippines	0.00	0.05	0.15
Singapore	0.05	0.00	0.29
Thailand	0.00	0.18	0.18

3.2 Sensitivity Analysis relative to the Threshold

The choice of the threshold is an arbitrary “rule of thumb”, aimed at detecting ‘enough’ crises: as the threshold increases, the crises detected are fewer and necessarily the worst ones. Since the outcome of this analysis is to classify periods, it is necessary to investigate what type of results would be ensuing. Table [3] reports the percent of periods identified as a crisis for each country as a function of the threshold τ . Several features of this table are worth pointing out: first and foremost, the inadequacy of the same parametric distribution as a reference distribution for the standardized

EMP (the first row reports the reference values of the probability to the right of the corresponding threshold value under a normal distribution).

Table [3] Relative incidence of crisis periods as a function of the threshold τ

Percent of episodes identified as crises	Values of threshold τ			
	1.5	2.0	2.5	3.0
Reference percentage when distribution is normal.	6.68	2.28	0.62	0.13
Developed	5.88	3.48	2.10	1.29
Denmark	7.78	4.90	2.59	0.86
Finland	5.76	3.46	2.31	1.44
France	5.76	3.46	2.31	1.73
Germany	6.34	2.02	0.86	0.58
Italy	5.19	4.03	2.59	2.31
Netherlands	5.76	4.03	2.31	1.73
Spain	4.90	3.46	2.59	1.73
Sweden	5.19	2.88	2.02	2.02
United Kingdom	7.34	3.54	1.52	0.51
Canada	6.33	3.80	2.53	1.27
Japan	4.30	2.78	1.52	0.25
Emerging	3.92	2.64	1.84	1.48
Argentina	5.06	3.80	2.53	2.53
Chile	3.04	1.77	1.52	1.01
Colombia	3.54	2.03	1.27	1.01
Mexico	5.57	3.04	2.78	2.03
Peru	3.04	2.03	1.27	1.27
Uruguay	3.29	2.03	0.76	0.51
Venezuela	3.29	3.04	2.28	2.03
Turkey	4.05	2.78	2.28	1.77
Indonesia	3.54	2.78	2.03	1.27
Korea	4.05	3.04	1.52	1.27
Malaysia	4.56	3.04	2.28	2.03
Philippines	4.56	2.53	2.28	1.77
Singapore	4.56	3.54	1.77	1.01
Thailand	2.78	1.52	1.27	1.27

Standardization is not enough to induce normality (especially for emerging countries): most empirical distributions are positively skewed (20 out of the 25 countries), and the kurtosis is above 3 for all countries. If one chooses a fixed threshold, Table [3] says that for lower levels of τ , a larger number of crises are signalled for developed than for emerging countries, while the situation reverses as τ increases (pointing to fatter tails for the emerging countries). With the value adopted by ERW, $\tau = 1.5$, for example, the percentage of periods identified as crises for developed and

emerging countries is 5.88 and 3.92 respectively, while when $\tau = 3.0$ we find 1.29 for developed and 1.48 percent for emerging countries. Even the ordering of countries within the sample relative to the number of crises detected changes with the threshold: Germany passes from the 3rd to the 23rd place when τ increases from 1.5 to 2.0; United Kingdom has the second highest incidence when the threshold is set at 1.5, while it has the second lowest when this moves to 3.0. The reverse happens to Mexico and Argentina.

The rate at which the number of crises detected for developed countries decreases with τ greatly outpaces the corresponding rate for emerging countries. This suggests that episodes of crisis detected for emerging countries correspond to a higher value of the (standardized) EMP index and, in a way, they are less sensitive to a fixed τ when this is large. This leads to the seemingly counterintuitive finding that developing countries tend to have a smaller number of crises when $\tau = 1.5$. The empirical evidence provided shows that the distributions of the EMP indexes are quite different from one another and calls into question the very definition of a crisis: it sounds less appropriate to use the same threshold to define a crisis for a country than it is to refer to country-specific characteristics.

Adopting a sort of Value at Risk framework, we may define as extreme periods worthy of attention those episodes which occur with at most a certain probability. The extension of a parametric EMP-based identification rule to a set of diverse countries suggests that, as already noted by Pontines and Siregar [2004b], we need to move towards alternative identifications rule that can better handle the sample distribution of the index. As we saw, Table [2] provides an illustrative example of how the binary crisis variable is sensitive to one of the three *fine tuning* issues that have been discussed in the previous subsections. But, to the best of our knowledge, no empirical study so far has tested the robustness of its results to changing definitions of crisis or to different choices

related to the underlying index¹⁵. Table [4] displays the 95th percentiles of the sample distributions of the standardized EMP index built according to eight different specifications as before.

Table [4]- 95th percentile standardized EMP with different specifications

	Logarithmic Differences				Exact Formula				Min	Max
	Constant		Time-varying		Constant		Time-varying			
	Gross	Net	Gross	Net	Gross	Net	Gross	Net		
Developed	1.61	1.61	1.67	1.66	1.61	1.61	1.67	1.66	1.61	1.67
Denmark	1.98	1.98	1.91	1.91	1.98	1.98	1.90	1.90	1.90	1.98
Finland	1.56	1.58	1.64	1.64	1.55	1.57	1.63	1.63	1.55	1.64
France	1.53	1.53	1.71	1.71	1.54	1.54	1.72	1.71	1.53	1.72
Germany	1.53	1.53	1.63	1.63	1.55	1.55	1.64	1.64	1.53	1.64
Italy	1.53	1.52	1.70	1.70	1.52	1.51	1.69	1.70	1.51	1.70
Netherlands	1.64	1.64	1.53	1.53	1.64	1.64	1.53	1.53	1.53	1.64
Spain	1.44	1.47	1.51	1.52	1.42	1.47	1.50	1.50	1.42	1.52
Sweden	1.53	1.53	1.71	1.71	1.52	1.52	1.70	1.70	1.52	1.71
United Kingdom	1.82	1.82	1.70	1.66	1.83	1.84	1.70	1.67	1.66	1.84
Canada	1.73	1.73	1.84	1.84	1.73	1.73	1.84	1.84	1.73	1.84
Japan	1.38	1.38	1.46	1.46	1.39	1.39	1.47	1.47	1.38	1.47
	Logarithmic Differences				Exact Formula				Min	Max
	Constant		Time-varying		Constant		Time-varying			
	Gross	Net	Gross	Net	Gross	Net	Gross	Net		
Emerging	1.35	1.35	1.34	1.34	1.28	1.29	1.27	1.31	1.27	1.35
Argentina	1.53	1.44	1.64	1.47	1.25	1.43	1.30	1.61	1.25	1.64
Chile	1.25	1.30	1.38	1.48	1.20	1.22	1.04	1.05	1.04	1.48
Colombia	1.32	1.32	1.28	1.28	1.32	1.32	1.32	1.32	1.28	1.32
Mexico	1.63	1.58	1.66	1.55	1.58	1.48	1.60	1.49	1.48	1.66
Peru	1.24	1.32	1.13	1.43	0.94	1.10	0.92	1.45	0.92	1.45
Uruguay	1.26	1.08	1.29	1.30	1.19	1.05	1.27	1.28	1.05	1.30
Venezuela	1.27	1.28	1.27	1.22	1.22	1.18	1.23	1.14	1.14	1.28
Turkey	1.39	1.40	1.25	1.29	1.28	1.29	1.19	1.27	1.19	1.40
Indonesia	1.24	1.20	1.25	1.05	1.21	1.07	1.25	1.05	1.05	1.25
Korea	1.32	1.48	1.16	1.35	1.30	1.44	1.18	1.39	1.16	1.48
Malaysia	1.41	1.41	1.46	1.46	1.41	1.40	1.46	1.46	1.40	1.46
Philippines	1.38	1.43	1.46	1.46	1.39	1.44	1.40	1.45	1.38	1.46
Singapore	1.49	1.49	1.42	1.42	1.49	1.49	1.42	1.42	1.42	1.49
Thailand	1.11	1.13	1.16	1.04	1.10	1.11	1.14	1.04	1.04	1.16

There are noticeable variations both by column (across countries) and by row (across specifications), although the averages by country subgroup are fairly stable, roughly 1.6 for developed countries and 1.3 emerging countries. Across countries we can mention a widespread range, e.g. in the first column between 1.38 for Japan and 1.98 for Denmark, and between 1.11 for

¹⁵ Because of lack of space we have not discussed an issue related with the data collection. As it is already pointed out by ERW, “monthly observations may not be a sufficiently fine periodicity to identify every speculative attack, especially unsuccessful ones.” An attack can be launched and concluded in a few days, and the behaviour of yearly averages of the relevant variables might fail to reveal it. As shown in the Table [2], several empirical works have used quarterly or monthly data.

Thailand and 1.63 for Mexico. Within countries the ranges are spread as well: Argentina and Korea, for example, have 95th percentiles in the ranges 1.25-1.64 and 1.16-1.48 respectively.

To complete the analysis, Table [5] displays the 99th percentiles of the sample distributions of the standardized EMP index. If the threshold were to be set at a higher value so that only 1% of the periods is classified as a crisis episode, the values increase considerably and more so for emerging countries than for developed ones (on average roughly 3.82 and 3.35 respectively). The variability across countries and specifications is maintained as well.

Table [5]-99th percentile standardized EMP with different specifications

	Logarithmic Differences				Exact Formula				Min	Max
	Constant		Time-varying		Constant		Time-varying			
	Gross	Net	Gross	Net	Gross	Net	Gross	Net		
Developed	3.37	3.38	3.31	3.31	3.41	3.41	3.35	3.34	3.31	3.41
Denmark	3.00	3.00	3.10	3.10	3.05	3.05	3.13	3.13	3.00	3.13
Finland	3.75	3.75	3.83	3.83	3.76	3.76	3.84	3.84	3.75	3.84
France	3.65	3.65	3.66	3.66	3.66	3.66	3.69	3.69	3.65	3.69
Germany	2.36	2.36	2.71	2.71	2.41	2.41	2.75	2.75	2.36	2.75
Italy	4.36	4.38	3.72	3.72	4.40	4.42	3.83	3.72	3.72	4.42
Netherlands	3.91	3.91	3.28	3.28	3.91	3.91	3.28	3.28	3.28	3.91
Spain	3.30	3.29	3.29	3.31	3.36	3.32	3.32	3.33	3.29	3.36
Sweden	4.10	4.10	3.51	3.51	4.17	4.17	3.66	3.66	3.51	4.17
United Kingdom	2.72	2.76	3.14	3.14	2.72	2.75	3.13	3.13	2.72	3.14
Canada	3.21	3.21	3.40	3.40	3.23	3.23	3.40	3.40	3.21	3.40
Japan	2.76	2.76	2.75	2.75	2.82	2.82	2.82	2.82	2.75	2.82
	Logarithmic Differences				Exact Formula				Min	Max
	Constant		Time-varying		Constant		Time-varying			
	Gross	Net	Gross	Net	Gross	Net	Gross	Net		
Emerging	3.80	3.88	3.71	3.85	3.76	3.81	3.81	3.93	3.71	3.93
Argentina	4.79	5.26	4.75	5.13	5.64	5.28	5.59	5.23	4.75	5.64
Chile	3.54	3.22	3.40	3.50	3.07	3.04	4.39	4.47	3.04	4.47
Colombia	3.11	3.11	2.96	2.96	3.16	3.16	3.06	3.06	2.96	3.16
Mexico	4.53	4.82	4.31	4.70	4.78	4.95	4.33	4.93	4.31	4.95
Peru	4.06	4.93	3.37	4.03	3.39	4.29	2.76	3.85	2.76	4.93
Uruguay	2.37	2.45	2.47	3.07	2.27	2.40	2.52	2.93	2.27	3.07
Venezuela	4.67	4.45	4.68	4.49	4.31	4.08	4.45	4.25	4.08	4.68
Turkey	4.59	4.50	3.92	3.55	4.51	4.52	3.88	3.59	3.55	4.59
Indonesia	3.37	3.51	3.74	3.85	3.32	3.44	3.64	3.74	3.32	3.85
Korea	3.18	3.50	3.13	3.72	3.06	3.39	3.07	3.68	3.06	3.72
Malaysia	3.67	3.66	4.09	4.09	3.74	3.73	4.20	4.20	3.66	4.20
Philippines	4.07	3.79	4.28	4.06	4.06	3.75	4.40	4.19	3.75	4.40
Singapore	3.11	3.11	2.81	2.81	3.12	3.12	2.82	2.82	2.81	3.12
Thailand	4.13	4.04	4.04	3.94	4.24	4.15	4.15	4.05	3.94	4.24

4. Reference country

In the construction of the EMP, a country is selected as a suitable benchmark in order to avoid reading the switches in the relevant domestic data driven by changes in the conditions prevailing on international markets as signals of pressure on the exchange market. The obvious benchmark for European countries has been Germany while the US is the reference country for the other OECD countries (see ERW). However when developing countries are involved, the choice is not as clear-cut. Pontines and Siregar [2004a] suggest, “hardly any studies have tested the sensitivity of this crisis index to the various possible choices of “the anchor” currencies”.

We argue that Pontines and Siregar [2004a] findings reveal just a specific case of a systematic impact of any shift in the reference country, as stated in Proposition [1]:

Proposition [1] –Given three countries, A, B and C, let us denote the EMP, say, for country A at time t with the reference country B, as EMP_t^{AB} . The following relation holds:

$$EMP_t^{AC} = EMP_t^{AB} + EMP_t^{BC} \quad \forall t \quad [2]$$

if the variations of the nominal exchange rate are computed as logarithmic differences, and the same weight is attached to each component of the EMP indices in [2].

Proof. [See the Appendix]

Under the hypothesis introduced in proposition [1], the EMP indices are characterized by a transitivity property the shift in the reference country is equivalent to adding the EMP between the two reference countries to the original index. Clearly the proposition is based on hypotheses that do not hold in actual applications, as the series are characterized by different volatility. Though, [2] can

be shown to provide a good approximation of the impact of a shift in the reference country even when the hypothesis of identical sets of weights is violated¹⁶.

Table [6] displays the changes in six Asian countries' crisis sets when Japan replaces the US in the role of reference country. The first (second) letter in the label of each column reflects whether the period is identified as a crisis, c, or tranquil, n, when Japan (the US) is the reference country.

Table [6] Effects of a change in the reference country for Asian countries

	cc	cn	nc	nn	Div
East Asian	11.0	5.2	8.8	370.0	0.56
Indonesia	13	3	0	379	0.19
Korea	12	4	6	373	0.45
Malaysia	12	6	16	361	0.65
Philippines	16	2	9	368	0.41
Singapore	5	13	16	361	0.85
Thailand	8	3	6	378	0.53

The extent of divergence of the two crisis sets is remarkable: on average the change in the reference country (From USA to Japan) determines a degree of divergence – computed as in [4] - equal to 0.56 between the two alternative crisis identification rules. In the case of Singapore, the two rules agree just in 5 out of the 34 periods that are identified as crisis by either of the two. For Malaysia and Thailand, the degree of divergence amounts to 0.65 and 0.53 respectively. The evidence provided thus confirms and extends the results advanced by Pontines and Siregar [2004a], and suggests that the choice of the appropriate anchor currency for East Asian countries is likely to have a significant bearing on the analysis of currency crisis determinants.

5. *Conclusions and directions for future research*

This paper attempted to provide a detailed description of the issues that arise from the adoption of the EMP index to study currency crises. The EMP represents a step forward from previous studies that relied on exchange rate movements alone to identify speculative pressures on a currency.

¹⁶ For brevity's sake we do not include this result (achieved using data from the seven Asian countries) in this paper. They are available upon request from the authors.

Particularly, it is widely used to study currency crises since it allows to signal those pressures on a currency that are softened or warded off through monetary authorities' interventions, thus avoiding a bias in the selection of crisis episodes due to the missing observation of unsuccessful speculative attacks, as it would happen if the selection rested merely on nominal exchange rate movements. However, it presents some problematic characteristics that deserve a thorough scrutiny.

Three fine tuning issues related to the index construction were reported, regarding: the weighting scheme adopted to combine the three underlying variables, the suitability of logarithmic differences to compute the percentage change of the exchange rate, and the theoretical issues that arise from the inclusion of net rather than gross reserves. Whenever possible, we have indicated possible alternatives, or the criteria that should inform the choice. Since the EMP, as we have rather extensively argued and showed, is notably sensitive to these seemingly minor choices, these should be explicitly stated and described, and the robustness of the results of following econometric analysis should be carefully assessed. Furthermore, we have shown that differences in the structure of the economic system, such as those observed in developing countries; strongly affect the consistency of the assumptions and, consequently, the results that can be achieved. Even the widely accepted use of logarithms can quite significantly distort the EMP index for these countries. This simple example is revealing of how an analytical tool that has been first created for OECD countries is not well suited for a straightforward application to developing countries. In addition evidence provided confirms that, since reference country may have a significant impact the analysis, a preliminary study is needed to evaluate the *appropriate anchor*.

Thus, an alternative EMP-based definition of crisis has to be still country-specific, but at the same time it has to satisfy two additional properties that do not belong to previous definitions. The properties that characterize the proposed definition are: 1) spatial relativity; 2) temporal relativity; 3) the classification of past observations does not depend on future data.

While the first property is shared by ERW definition, the other two represent an attempt to overcome two weaknesses of their crisis indicator. Indeed, the time invariance of the crisis threshold can constitute an undesirable feature if the EMP index presents some clusters of high volatility. These drive up the threshold, and render the index insensitive to those speculative pressures that occur during period of low volatility. Thus, instead of taking sample mean and standard deviation over the whole sample, moving average and standard deviation can be employed, computed over a period that is long enough to detect structural breaks in volatility from periods of increasing pressure. However, this possible approach suffers from two main shortcomings: first, it shrinks the sample size and information is lost, as a centred moving average is a function of lags and leads and thus missing values are generated for the initial and final observations. Second, it is not consistent with the third property, as the classification of an observation is still dependent on future values.

At least two reasons can be advanced to justify the desirability of the independence of the identification of past observations from future data, a property that is satisfied by the definitions employed by Frankel and Rose [1996] and Kumar et al. [1998]. First, an instrumental one, as the dependence of past identifications on future observations gives rise to a never-ending process of data revision, while a second reason relates to the analytical perspective from which currency crises are observed. While subsequent data are useful to put the conditions prevailing on the exchange market in an historical perspective, these should not be relevant in a more policy-oriented approach, that is more interested on the evaluation that economic agents give of current events.

This paper suggests that existing EMP-based crisis indicators may not be well suited for the study of currency crises in developing countries, as they lead to a questionable selection of crisis episodes, and suggest that developing countries are much less crisis-prone than developed countries. In order to test the actual relevance of this intuition, we intend to deepen this research in two complementary directions: first, to understand *punctually* and clarify the differences between ERW

index of crises and our proposed crisis selection rules; second, to build different crisis sets according to diverging crisis selection rule, and to employ these sets as regressands in an econometric analysis – either a logit or a probit model - on currency crises determinants.

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Appendix

A1 – Outline of the dataset

We have collected monthly data for a sample of 26 countries over the period 1970-2002. The sample is made up of ten European countries (Denmark, Finland, France, Germany, Italy, the Netherlands, Spain, Sweden, Turkey and the United Kingdom), seven Latin American (Argentina, Chile, Colombia, Mexico, Peru, Uruguay and Venezuela), seven Asian (Indonesia, Japan, Malaysia, Philippines, Singapore, South Korea and Thailand), the United States of America and Canada. For the European countries that have adopted the euro, the series are interrupted in December 1998.

The data source is given by IMF’s International Financial Statistics, IFS. The data include:

- monthly average of the nominal exchange rates, line rf IFS, with three possible reference countries (Germany, Japan, US);

- short term deposit interest rate, line 60b IFS;¹⁷
- total reserves excluding gold, line 11.d IFS;
- restricted money base, line 34 IFS;
- IMF credit and loans outstanding, line 2tl IFS.

If not otherwise specified, the EMP is computed – in accordance with ERW – under the following default choices: the US is the reference country, the series are weighted by the reciprocal of their sample standard deviation, exchange rate variations are computed as logarithmic differences, reserves are considered gross of the liabilities towards the IMF. A crisis is detected whenever the EMP is above its mean plus 1.5 times its standard deviation.

A2 – Contraction Mappings and variance of the transformed distribution

A function g is said to be a Lipschitz function over the domain $D \subseteq \mathbb{R}$, if it exists $c > 0$ such that:

$$|g(x) - g(y)| \leq c|x - y| \text{ for every } x, y \in D \subseteq \mathbb{R}.$$

If $0 < c < 1$, then g is said to be a contraction mapping over the domain D .

We want to show that the variance of a distribution X belonging to D is greater than the variance of the distribution that is obtained applying the function g to X . The above definition of a contraction mapping implies that:

$$|g(X) - g[E(X)]| \leq c|X - E(X)|$$

¹⁷ If unavailable, we have the three months lending rate; for the US, we have used the Treasury Bill Rate.

$$\{g(X) - g[E(X)]\}^2 \leq c^2 [X - E(X)]^2$$

Taking the expected value of both sides, we obtain:

$$E\{\{g(X) - g[E(X)]\}^2\} \leq c^2 E\{[X - E(X)]^2\}$$

The r.h.s. is the variance of the X distribution, $V(X)$. The l.h.s. can be rewritten adding and subtracting the mean of the transformed distribution, i.e. $E[g(x)]$, within the inner parenthesis.

$$\begin{aligned} E\{\{g(X) - g[E(X)]\}^2\} &= E\{\{g(X) - E[g(x)] + E[g(x)] - g[E(X)]\}^2\} \\ &= E\{\{[g(X) - E[g(X)]]\}^2\} + E\{\{E[g(X)] - g[E(X)]\}^2\} + \\ &\quad + 2E\{\{g(x) - E[g(x)]\} * E\{g(x) - g[E(x)]\}\} \\ &= V[g(X)] + \{E[g(X)] - g[E(X)]\}^2 \end{aligned}$$

as $E\{\{g(x) - E[g(x)]\} * E\{g(x) - g[E(x)]\}\} = 0$ and $E\{\{g(X) - E[g(X)]\}^2\} = V[g(x)]$, the variance of the transformed distribution. This implies:

$$V[g(X)] + \{E[g(X)] - g[E(X)]\}^2 \leq c^2 V(X)$$

that is:

$$V[g(X)] \leq c^2 V(X) - \{E[g(X)] - g[E(X)]\}^2 \leq c^2 V(X)$$

As g is a contraction mapping, i.e. $0 < c < 1$, then $V[g(x)] < V(X)$. Q.E.D.

A3 - Proof of Proposition [1]

According to the definition of exchange market pressure, and assuming an identical set of weights in the three indexes, we have:

$$EMP_{AC} = \alpha \left(\frac{\Delta E^{AC}_t}{E^{AC}_t} \right) + \beta \Delta(i^A_t - i^C_t) + \gamma \Delta(r^C_t - r^A_t)$$

$$EMP_{AB} = \alpha \left(\frac{\Delta E^{AB}_t}{E^{AB}_t} \right) + \beta \Delta(i^A_t - i^B_t) + \gamma \Delta(r^B_t - r^A_t)$$

$$EMP_{BC} = \alpha \left(\frac{\Delta E^{BC}_t}{E^{BC}_t} \right) + \beta \Delta(i^B_t - i^C_t) + \gamma \Delta(r^C_t - r^B_t)$$

Where the superscripts denote the country to which the variable is referred, and E^{AB} is the value of currency B expressed in units of currency A. We want to show that the hypothesis of identity of weights implies that the index satisfies a transitive property, that is to say $EMP_{AC} = EMP_{AB} + EMP_{BC}$. The r.h.s of this identity is given by:

$$EMP_{AB} + EMP_{BC} = \alpha \left(\frac{\Delta E^{AB}_t}{E^{AB}_t} \right) + \beta \Delta(i^A_t - i^B_t) + \gamma \Delta(r^B_t - r^A_t) + \alpha \left(\frac{\Delta E^{BC}_t}{E^{BC}_t} \right) + \beta \Delta(i^B_t - i^C_t) + \gamma \Delta(r^C_t - r^B_t)$$

$$EMP_{AB} + EMP_{BC} = \alpha \left[\left(\frac{\Delta E^{AB}_t}{E^{AB}_t} \right) + \left(\frac{\Delta E^{AC}_t}{E^{AC}_t} \right) \right] + \beta \Delta(i^A_t - i^C_t) + \gamma \Delta(r^C_t - r^A_t)$$

As we have assumed that $\left(\frac{\Delta E^{AB}_t}{E^{AB}_t} \right) = \ln \left(\frac{E^{AB}_t}{E^{AB}_{t-1}} \right)$, we have:

$$\alpha \left[\left(\frac{\Delta E^{AB}_t}{E^{AB}_t} \right) + \left(\frac{\Delta E^{AC}_t}{E^{AC}_t} \right) \right] = \alpha \left[\ln \left(\frac{E^{AB}_t}{E^{AB}_{t-1}} \right) + \ln \left(\frac{E^{AC}_t}{E^{AC}_{t-1}} \right) \right]$$

$$\begin{aligned}
&= \alpha \ln \left[\left(\frac{E^{AB}_t}{E^{AB}_{t-1}} \right) \left(\frac{E^{AC}_t}{E^{AC}_{t-1}} \right) \right] \\
&= \alpha \ln \left[\frac{E^{AB}_t * E^{AC}_t}{E^{AB}_{t-1} * E^{AC}_{t-1}} \right] \\
&= \alpha \ln \left(\frac{E^{AC}_t}{E^{AC}_{t-1}} \right) = \alpha \left(\frac{\Delta E^{AC}_t}{E^{AC}_t} \right)
\end{aligned}$$

As $E^{AB}_t * E^{BC}_t = E^{AC}_t$. Hence:

$$EMP_{AB} + EMP_{BC} = \alpha \left(\frac{\Delta E^{AC}_t}{E^{AC}_t} \right) + \beta \Delta(i^A_t - i^C_t) + \gamma \Delta(r^C_t - r^A_t) = EMP_{AC} \text{ Q.E.D.}$$

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